The complete relativistic kinetic model of symmetry violation in isotopic expanding plasma. III. Specific entropy calculation.

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Abstract

A complete model of baryon production in an expanding, primordially symmetric hot Universe is constructed in the framework of general-relativistic kinetic theory. In this model specific model for a baryon is calculated and graphs of the value dependence are constructed.

1 Transformation to the dimentionless variables

In the previous papers of the authors [1], [2] in terms of the developed relativistic kinetic theory of baryons production[1] an expression for final concentration of baryons in hot Universe was obtained (23)[2], using which along with the dense entropy equation (26)[2], we get the sought-for expression for final dense of specific entropy accounted for one baryon

$$\delta_S = \frac{N_b(\infty)}{S} = \frac{30\Delta r \mathcal{N}_X}{\pi^4 \mathcal{T}^3 \mathcal{N}} \int\limits_0^\infty \exp\left(-\int\limits_t^\infty \Psi(t')dt'\right) G(t)dt, \tag{1}$$

where

$$\Psi(t) = \frac{2N_X}{\pi^2 \mathcal{T}^3} \int\limits_0^\infty \mathbb{P}^2 \dot{\Phi} f_0 \beta_0 d\mathbb{P}, \tag{2}$$

$$G(t) = \frac{1}{\pi^2} \int\limits_0^\infty \mathbb{P}^2 \dot{\Phi} \delta f d\mathbb{P}; \tag{3}$$

derivatives by time are denoted by a dot t, δf - is a deviation of the boson distribution function from equilibrium in a symmetric plasma $\lambda = 0$

$$\delta f(\mathbb{P}, t) = -e^{-\Phi(\mathbb{P},t)} \int\limits_0^t e^{\Phi(\mathbb{P},t')} \dot{f}_0(0; \mathbb{P}, t')dt', \tag{4}$$

and the following designation is used

$$\Phi(\mathbb{P}, t) = \frac{1}{\tau_0} \int\limits_0^t \frac{a(t')\beta_0(\mathbb{P}, t')dt'}{\sqrt{a^2(t') + \mathbb{P}^2/m_X^2}}. \tag{5}$$

[1]As well as the other similar particles arising as a result of thermodynamic equilibrium violation and spontaneous CP invariance violation.

According to the formulae (28)-(35)[2] we come from the temporal variable t and the impulse one $\mathbb{P}$ to the dimentionless temporal variable η and the impulse variable ξ

$$t = \tau_0 \eta. \tag{6}$$

In doing so we choose the normalization of scale factor for a completely ultra-relativistic stage of universe expansion, such one that

$$a(t) = \sqrt{\tau_0 \eta}, \tag{7}$$

where τ_0 is the supermassive boson decay time in its own frame of reference

$$\tau_0 = \frac{4\pi m_X}{s^2} \sim \frac{3}{2}(m_X \alpha)^{-1}. \tag{8}$$

and introduce a dimentionless parameter σ

$$\sigma = \frac{m_X}{T(\tau_0)} = \frac{m_X\sqrt{\tau_0}}{\mathcal{T}_0} = \frac{\chi\sqrt{m_X}}{\sqrt{\alpha}\mathcal{T}_0}, \tag{9}$$

The dimentionless impulse variable ξ let us introduce with the help of the relation (Ref. [2]):

$$\mathbb{P} = m_x\sqrt{\tau_0}\xi \Rightarrow \xi = \frac{\mathbb{P}}{m_X\sqrt{\tau_0}} \tag{10}$$

so that

$$\frac{E}{T} = \sqrt{m_X^2 + \mathbb{P}^2/a^2(t)}T = \sigma\sqrt{\eta + \xi^2} \tag{11}$$

and the equilibrium function of the supermassive bosons is equal to (Ref. (13)[2])

$$f_0(\eta, \xi) = \frac{1}{e^{\sigma\sqrt{\eta+\xi^2}} - 1}. \tag{12}$$

As the done investigations have shown the final results of the investigations are very weakly sensitive to statistic factors consideration by the factors $\Phi(\eta, \xi)$ calculating, whereas they are very sensitive to the static factors consideration at the other stages of calculation. Therefore further on we shall calculate the function η, ξ in the Boltzmann approximation , while at the other stages of calculation we shall hold the statistic factors. Then in the Boltzmann approximation $\beta_0 \approx 1/2$, and we obtain for the function Φ:

$$\dot{\Phi}(\mathbb{P}, t) = \frac{1}{2\tau_0}\frac{\sqrt{\eta}}{\sqrt{\eta + \xi^2}} \tag{13}$$

and -

$$\Phi(\xi, \eta) = \frac{1}{2}\left(\sqrt{\eta}\sqrt{\eta + \xi^2} - \xi^2 \ln\frac{\sqrt{\eta} + \sqrt{\eta + \xi^2}}{\xi}\right). \tag{14}$$

In consequence of (13) $G(\eta, \xi)$ is a nonnegative monotonically increasing function

$$\Phi(\eta, \xi) \geq 0; \quad \frac{\partial\Phi(\eta, \xi)}{\partial\eta} = \frac{1}{2}\frac{\sqrt{\eta}}{\sqrt{\eta + \xi^2}} \geq 0, \tag{15}$$

with

$$\lim_{\eta\to 0}\Phi(\eta,\xi)=0. \tag{16}$$

Note that the rather large expression for the function $\Phi(\xi,\eta)$, obtained in [2], can be reduced to the given above after simple transformations.

Thus coming to the new variables we get an expression for specific entropy

$$\delta_S=\frac{15\Delta r\mathcal{N}_X}{2\pi^6\mathcal{N}}\sigma^3\int\limits_0^\infty d\eta e^{-\Theta(\eta)}\sqrt{\eta}\int\limits_0^\infty\frac{\xi^2 d\xi}{\sqrt{\eta+\xi^2}}\delta f(\eta,\xi), \tag{17}$$

where

$$\Theta(\eta)=\int\limits_t^\infty\Psi(t)dt.$$

As far as $\dot{\Phi}\geq 0$, then $\Psi(t)>0$, thus $\Theta(\eta)$ is a nonnegative monotonically decreasing function

$$\frac{d\Theta}{d\eta}\leq 0. \tag{18}$$

In this case

$$\delta f(\eta,\xi)=e^{-\Phi(\eta,\xi)}\int\limits_0^\eta d\eta' e^{\Phi(\eta',\xi)}\frac{\partial}{\partial\eta'}\frac{1}{e^{\sigma\sqrt{\eta'+\xi^2}}-1}. \tag{19}$$

2 Functions $\Psi(t)$ and $\Theta(t)$

Coming to the new variables in Boltzmann approximation of the function $\Phi(x)$ (14) the expression for the function $\Psi(\eta)$, in which the statistic factor is already taken into consideration, is

$$\Psi(\eta)=\frac{\sqrt{\eta}N_X\sigma^3}{2\pi^2\tau_0}\int\limits_0^\infty\frac{1}{e^{\sigma\sqrt{\eta+\xi^2}}-1}\frac{\xi^2 d\xi}{\sqrt{\eta+\xi^2}} \tag{20}$$

Let us introduce new variables x and z

$$\xi=\sqrt{\eta}\sinh(x),\quad z=\sigma\sqrt{\eta}. \tag{21}$$

Then we get

$$\Psi(\eta)=\frac{N_X}{\pi^2\tau_0}z^3\int\limits_0^\infty\frac{\sinh^2 t dt}{e^{z\cosh t}-1}.$$

Calculating the integral $\int\Psi dt$ and changing the order of integration in the obtained expression we arrive at

$$\Theta(\eta)=\int\limits_t^\infty\Psi(t')dt'=\frac{2N_X}{\pi^2\sigma^2}\int\limits_0^\infty\frac{\sinh^2 x}{\cosh^5 x}dx\int\limits_{\sqrt{\eta}\sigma\cosh x}^\infty\frac{\nu^3}{e^\nu-1}d\nu. \tag{22}$$

In particular integrating over the whole interval of the value t we obtain the integrals product

$$\Theta(0) = \int\limits_0^\infty \Psi(t')dt' = \frac{2N_X}{\pi^2\sigma^2} \int\limits_0^\infty \frac{\sinh^2 x}{\cosh^5 x} dx \int\limits_0^\infty \frac{\nu^3}{e^\nu - 1} d\nu,$$

one of them is expressed in terms of ζ - Riemann function

$$\int\limits_0^\infty \frac{\nu^3}{e^\nu - 1} = \frac{\pi^4}{15},$$

and the other is easily calculated

$$\int\limits_0^\infty \frac{\sinh^2 x}{\cosh^5 x} dx = \frac{\pi}{16}.$$

As a result we find

$$\Theta(0) = \int\limits_0^\infty \Psi(t')dt' = \frac{\pi^3 N_X}{120\sigma^2}. \tag{23}$$

Thus we can write down

$$\Theta(\eta) = \Theta(0) - \frac{2N_X}{\pi^2\sigma^2} \int\limits_0^\infty \frac{\sinh^2 x}{\cosh^5 x} dx \int\limits_0^{\sqrt{\eta}\sigma \cosh x} \frac{\nu^3}{e^\nu - 1} d\nu.$$

The inner integral can be expressed in terms of the function[2]

$$D(x) = \frac{3}{x^3} \int\limits_0^x \frac{t^3}{e^t - 1} dt.$$

the function $D(x)$ has the following asymptotics

$$D(x) \approx \begin{cases} 3\sum\limits_0^\infty \dfrac{B_n}{(n+3)n!}\, x^n, & x \lesssim 1; \\ \\ \dfrac{\pi^4}{5x^3} - 3\left(1 + \dfrac{3}{x} + \dfrac{6}{x^2}\right) e^{-x}, & x \gg 1, \end{cases} \tag{24}$$

where B_n are Bernoulli numbers. The function $D(X)$ graph is shown in Fig. 1.

[2]The function $D(x)$ is connected with the Debye functions (Ref., for example [3]).

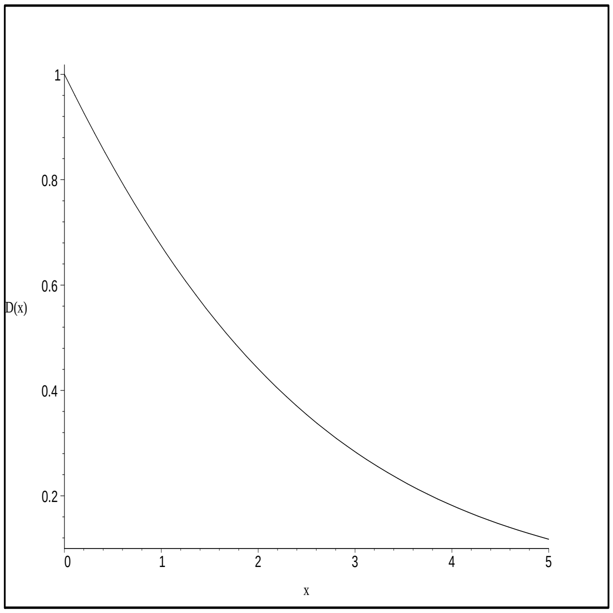

Fig.**1.** *The function $D(x)$ graph constructed with the help of the Maple package on the approximations of the type (24) .*

Thus finally we obtain

$$\Theta(\eta) = \frac{\pi^3 N_X}{120\sigma^2}\Xi(\sqrt{\eta}\sigma), \tag{25}$$

where a monotonically decreasing function $\Xi(x)$ is introduced

$$\Xi(x) = 1 - \frac{80}{\pi^5}x^3 \int\limits_0^\infty \tanh^2 x D(x\cosh z)dz, \tag{26}$$

varying in the interval

$$0 \geq \Xi(x) \leq 1.$$

To calculate with the functions $D(x)$ and $\Xi(x)$ a special library in the package of symbol Maple Mathematics was worked out in which the proceeders of time-optimal calculation of these functions with the help of different approximations are determined. The function $\Xi(x)$ graph is shown in Fig. 2

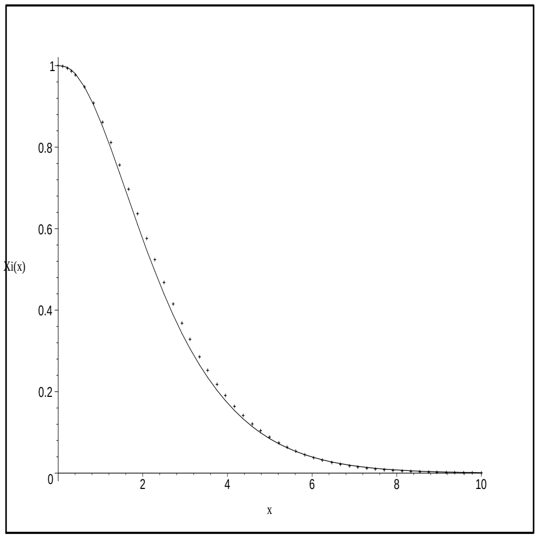

Fig.**2.** *The function $\Xi(x)$ graph. The dotted line denotes the extrapolating function $F(x)$ graph (27).*

In the interval $[0, 10]$ the function $\Xi(x)$ is well extrapolated by the function

$$F(x) = \frac{e^{-\alpha x^2}}{1 + \beta x^2} \tag{27}$$

with the parameters $\alpha = 0,05$ and $\beta = 0,09$.

3 Equilibrium deviation $\delta f(\eta, \xi)$

Now let us calculate the function $\delta f(\eta, \xi)$. As it is not difficult to see the function $\Phi(\eta, \xi)$ is a slowly varying function, because in consequence (15)

$$\Phi'_\eta < 1/2,$$

while

$$\lim_{\eta \to 0} e^{\Phi(\eta,\xi)} = 1,$$

and in wide limits of changing the variables η, ξ:

$$\exp(\Phi(\eta, \xi)) \approx 1.$$

And the derivative of the equilibrium function of distribution has the order

$$\frac{f_0'}{f_0} = -\frac{\sigma}{2\sqrt{\eta + \xi^2}} \frac{e^{\sigma\sqrt{\eta+\xi^2}}}{e^{\sigma\sqrt{\eta+\xi^2}} - 1} \tag{28}$$

and infinitely grows in the range $\sigma\sqrt{\eta+\xi^2}\to 0$; By large values of this argument this variable is small. Thus, values of the variables η, ξ in the range $\sqrt{\eta+\xi^2}\lesssim \sigma^{-1}$ in which $\exp(\Phi)$ can be considered approximately constant make the main contribution to the distribution function deviation from equilibrium. Thus, integrating by parts in (19) we find approximately

$$\delta f(\eta,\xi)\simeq \frac{1}{e^{\sigma\xi}-1}-\frac{e^{-\Phi(\eta,\xi)}}{e^{\sigma\sqrt{\eta+\xi^2}}-1}. \tag{29}$$

Substituting this function into the inner integral (17) and introducing the function

$$\mathrm{Df}(\eta,\sigma)=\sigma^2\int\limits_0^\infty \frac{\xi^2 d\xi}{\sqrt{\eta+\xi^2}}\left[\frac{1}{e^{\sigma\xi}-1}-\frac{e^{-\Phi(\eta,\xi)}}{e^{\sigma\sqrt{\eta+\xi^2}}-1}\right] \tag{30}$$

we obtain

$$\delta_S=\frac{15\Delta r\mathcal{N}_X}{2\pi^6\mathcal{N}}\sigma\int\limits_0^\infty d\eta e^{-\Theta(\eta)}\sqrt{\eta}\mathrm{Df}(\eta,\sigma). \tag{31}$$

Note, the introduced earlier function $\mathrm{Df}(\eta,\sigma)$ is proportional to the perturbed track of tensor of X - bosons energy-impulse

$$\delta T_X=g_{ik}\delta T_X^{ik}=m_X^2\int\limits_{P(X)}\delta f_X dP.$$

Coming to the numerical integration in (31) let us note that the inconvenient for a numerical integration integrals are just in in the function $\mathrm{Df}(\eta,\sigma)$, so the direct application of numerical integration runs against the divergence problem. Therefore, at first the integral (30) is necessary to be transformed into a convenient for a numerical integration type. For this let us rewrite the integral (30) in the equivalent form

$$\mathrm{Df}(\eta,\sigma)=\sigma^2\left[\int\limits_0^\infty \frac{\xi^2 d\xi}{\sqrt{\eta+\xi^2}}\left(\frac{1}{e^{\sigma\xi}-1}-\frac{1}{e^{\sigma\sqrt{\eta+\xi^2}}-1}\right)+\right.$$

$$\left.+\int\limits_0^\infty \frac{\xi^2 d\xi}{\sqrt{\eta+\xi^2}}\frac{1-e^{-\Phi(\eta,\xi)}}{e^{\sigma\sqrt{\eta+\xi^2}}-1}\right].$$

Let us study the first part of the integral

$$A=\int\limits_0^\infty \frac{\xi^2 d\xi}{\sqrt{\eta+\xi^2}}\left(\frac{1}{e^{\sigma\xi}-1}-\frac{1}{e^{\sigma\sqrt{\eta+\xi^2}}-1}\right)\equiv$$

$$\int\limits_0^\infty \frac{\xi^2 d\xi}{\sqrt{\eta+\xi^2}}\frac{1}{e^{\sigma\xi}-1}-\int\limits_0^\infty \frac{\xi^2 d\xi}{\sqrt{\eta+\xi^2}}\frac{1}{e^{\sigma\sqrt{\eta+\xi^2}}-1}.$$

in the first integral we make a substitution $\xi = \sqrt{\eta}x$, and in the second $\xi = \sqrt{x^2-1}$. Then we get

$$A = \eta \left[\int_0^\infty \frac{x^2 dx}{\sqrt{1+x^2}} \frac{1}{e^{\sigma\sqrt{\eta}x}-1} - \int_1^\infty \frac{\sqrt{\eta+x^2}dx}{e^{\sigma\sqrt{\eta}x}-1} \right].$$

Bringing these integrals together we obtain

$$A = \eta \left[\int_0^1 \frac{x^2 dx}{\sqrt{1+x^2}} \frac{1}{e^{\sigma\sqrt{\eta}x}-1} + \int_1^\infty \frac{dx}{\sqrt{\eta+x^2}} \frac{1}{e^{\sigma\sqrt{\eta}x}-1} \right].$$

Now we transform the B part of the integral

$$B = \int_0^\infty \frac{\xi^2 d\xi}{\sqrt{\eta+\xi^2}} \frac{1-e^{-\Phi(\eta,\xi)}}{e^{\sigma\sqrt{\eta+\xi^2}}-1}.$$

Substituting the expression for $\Phi(\eta,\xi)$ into the integral and making a substitution $\xi = \sqrt{\eta}x$, we transform this integral

$$B = \eta \int_0^\infty \frac{x^2 dx}{\sqrt{1+x^2}} \frac{1-e^{-\frac{1}{2}\eta\left(\sqrt{1+x^2}-x^2\ln\frac{1+\sqrt{1+x^2}}{x}\right)}}{e^{\sigma\sqrt{\eta}\sqrt{1+x^2}}-1}.$$

In this integral the difficulties of numerical integration arise under the conditions of $\sigma\sqrt{\eta}\to 0$. Expanding the exponent into Tailor series small values η, we get approximately

$$B \approx \frac{1}{2}\eta^2 \int_0^\infty \frac{x^2 dx}{\sqrt{1+x^2}} \frac{\sqrt{1+x^2}-x^2\ln\frac{1+\sqrt{1+x^2}}{x}}{e^{\sigma\sqrt{\eta}\sqrt{1+x^2}}-1}.$$

Numerical integration of these expression does not run to any difficulties already. Considering the above comments in the Maple package there was created a library of special procedures of rapid calculation of the function $\mathrm{Df}(\eta,\sigma)$ for any values of variables. The graphs of the functions $\mathrm{Df}(\eta,\sigma)$ obtained with the help of these procedures are shown in Fig. 3.

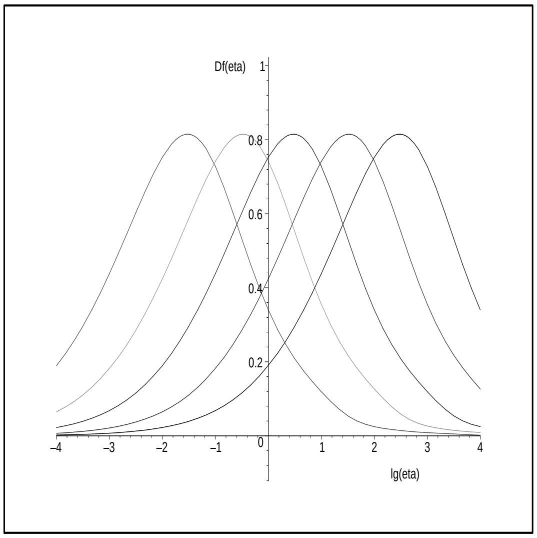

Fig.**3.** *Functions* $\mathrm{Df}(\eta, \sigma)$ *subjecting to the parameter* σ. *Along the abscissa axis* $\lg \eta$ *is plotted. From the left to the right* $\sigma = 10$, $\sigma = 3$, $\sigma = 1$, $\sigma = 0,3$, $\sigma = 0,1$. .

4 Results

Before proceeding to the results presentation let us carry out a convenient function δ_S normalization. As noted in [1], in the papers [4]-[6] the specific entropy estimation for one baryon was obtained (formula (5) [1]):

$$\delta_S^0 = \frac{45\zeta(3)}{4\pi^4} \frac{N_X}{N} \Delta r. \tag{32}$$

Therefore we shall correspond our results to this estimation by introducing a relative variable

$$\Delta_S = \frac{\delta_S}{\delta_S^0} \tag{33}$$

so called the reduced specific entropy. Carrying out numerical integration in the expression (31) with the help of the given procedures in the Maple package we obtain a graphs family of the function $\Delta_S(\sigma)$. In Fig. 4 we show calculated graphs of dependence $\Delta_S(\sigma)$ by different values of N_X - number of $X-$ boson types which is a parameter of the model of interactions.

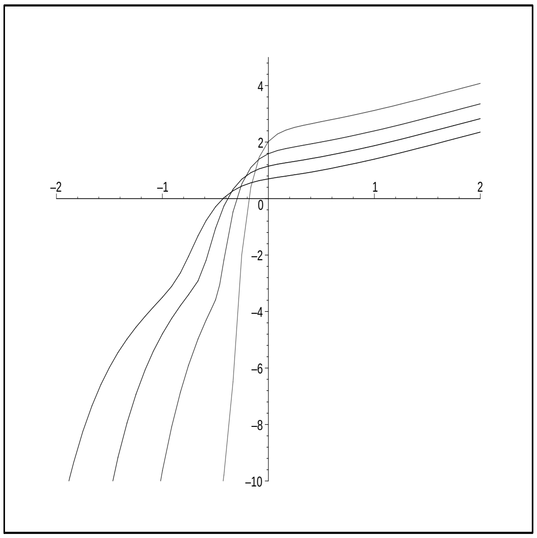

Fig.**4.** *The given specific entropy for one baryon,* $\lg \Delta_S(\sigma, N_X)$*, subjecting to the number of* X*-bosons,* N_X*. Along abscissa axis the values of* $\lg \sigma$ *are plotted. In the right part of the figure from bottom up* $N_X = 1$*;* $N_X = 3$*;* $N_X = 10$*;* $N_X = 53$. .

Passing on the analysis of the results we firstly note that the calculations carried out in terms of complete kinetic theory Showed a sufficient dependence of the produced baryon charge in the quantity of X - bosons types. Let us point out the following general tendency of this dependence of the produced baryon charge: at $\sigma \lesssim 0,4 \div 0,8$ with the increase of the number of X - bosons types the given specific entropy increases , and at $\sigma \gtrsim 0,4 \div 0,8$ on the contrary it decreases, moreover in the range of small values of the parameter σ the dependence of the given entropy on N_X is especially perceptible. At the same time we should remember that the absolute value of specific entropy equals to

$$\delta_S = \Delta_S \delta_S^0 = \Delta_S \frac{45\zeta(3)}{4\pi^4} \frac{N_X}{N} \Delta r. \tag{34}$$

On the other hand we can suppose the factor N_X/N (relation of the number of X-bosons types to the general number of particles types) does not strongly depend on the field model of interactions, therefore the conclusion concerning dependence of the number of X - bosons types of the given entropy can be carefully transferred to the absolute value of specific entropy also. These peculiarities of dependence of specific entropy on the number of X - bosons types are shown in Fig. 5.

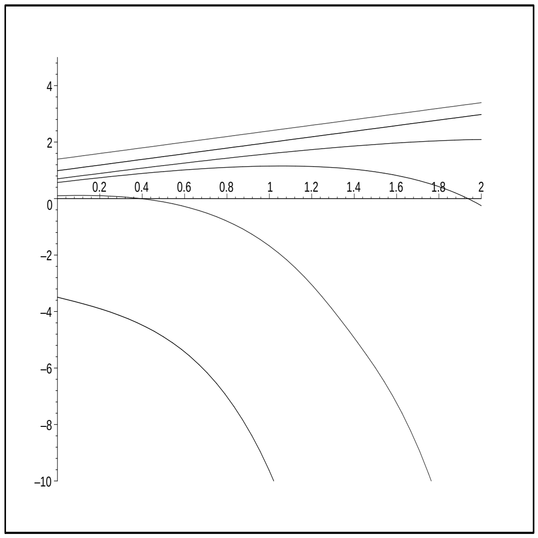

Fig.**5.** *Dependence of the given specific entropy for one baryon on the number of X - bosons types. Bottom-up* $\sigma = 0,1$*;* $\sigma = 0,4$*;* $\sigma = 0,7$*;* $\sigma = 1$*;* $\sigma = 3$*;* $\sigma = 10$*. Along the abscissa axis the values of* $\lg N_X$ *are plotted, along the ordinate axis the values of* $\lg \Delta_S$ *are..*

Further, the value (32) obtained by a number of authors, in the kinetic theory is reached at the values of the parameter $\sigma = 0,4 \div 0,6$. Moreover the kinetic model of cosmological baryogenesis detected a finer structure of this process than the one which was produced by the hydrodynamic theory of this process developed earlier in the quoted papers [7]-[9]. The difference of our results from the quoted papers results, especially in the range of small values of the parameter σ, is caused by essential influence of nonequilibrium processes on the final result in this range. It is not difficult to see that it is this range where the function of X-bosons distribution most differs from the equilibrium one. Of course this fact cannot be taken into consideration in the hydrodynamic model of bariogenesis effectively.

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